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Tracing CP-violation in Lepton Flavor Violating muon decays

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Abstract: Although the Lepton Flavor Violating (LFV) decay $\mu^+ \to e^+\gamma$ is forbidden in the Standard Model (SM), it can take place within various theories beyond the SM. If the branching ratio of this decay saturates its present bound [i.e., $\mathrm{Br}(\mu^+ \to e^+\gamma) \sim 10^{-11}$], the forthcoming experiments can measure the branching ratio with high precision and consequently yield information on the sources of LFV. In this paper, we show that for polarized μ^+, by studying the angular distribution of the transversely polarized positron and linearly polarized photon we can derive information on the CP-violating sources beyond those in the SM. We also study the angular distribution of the final particles in the decay $\mu^+ \to e_1^+ e^- e_2^+$ where e_1^+ is defined to be the more energetic positron. We show that transversely polarized e_1^+ can provide information on a certain combination of the CP-violating phases of the underlying theory which would be lost by averaging over the spin of e_1^+.

Keywords: Rare Decays, Beyond Standard Model, CP violation.

Contents

1. Introduction

While the Standard Model (SM) preserves the lepton flavor, its various extensions such as supersymmetry or large extra dimensions can lead to Lepton Flavor Violating (LFV) rare decays $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^-e^+$ detectable in the forthcoming experiments. The present experimental bounds on the branching ratios of these processes are [1]

$$\mathrm{Br}(\mu^+ \to e^+\gamma) < 1.2 \times 10^{-11} \quad \mathrm{Br}(\mu^+ \to e^+e^+e^-) < 1.0 \times 10^{-12} \text{ at 90\% C.L.}$$

The MEG experiment at PSI [2], which is under construction, will be able to probe $\mathrm{Br}(\mu^+ \to e^+\gamma)$ down to 10^{-14}. Thus, if this branching ratio saturates the present bound (i.e., $\mathrm{Br}(\mu^+ \to e^+\gamma) \sim 10^{-11}$) the future searches will enjoy high statistics and can make precise measurement limited only by systematics. Moreover, since the muons are produced by decay of stopped pions (at rest), they will be almost 100% polarized. Thus, studying the angular distribution of the final positrons, we can learn about phenomena such as parity violation, through which more information on the sources of LFV can be extracted [3]. Among the various extensions of the SM that can give rise to lepton flavor violating phenomena, in the literature the Minimal Supersymmetric Standard Model (MSSM) and large extra dimensions have received particular attention. It is well-known that in both cases integrating out the heavy states of the model, the LFV Lagrangian responsible for $\mu \to e\gamma$ can be written as

$$\mathcal{L} = A_R \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + A_L \bar{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu} + A_R^* \bar{e}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu} + A_L^* \bar{e}_R \sigma^{\mu\nu} \mu_L F_{\mu\nu} \tag{1.1}$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ and $F_{\mu\nu}$ is the photon field strength: $F_{\mu\nu} = \partial_\mu \varepsilon_\nu - \partial_\nu \varepsilon_\mu$. Although Lagrangian in eq. (1.1) is not the most general form of the effective LFV Lagrangian, throughout this paper we consider only these terms. In the appendix, we consider a more general form of the effective Lagrangian and show that (1.1) is indeed the dominant part.

It can be shown that averaging over the spins of the final particles, Lagrangian (1.1) yields [4]

$$\frac{d\Gamma(\mu^+ \to e^+\gamma)}{d\cos\theta} = \frac{1}{8\pi} m_\mu^3 \left[|A_R|^2 (1 - \mathbb{P}_\mu \cos\theta) + |A_L|^2 (1 + \mathbb{P}_\mu \cos\theta) \right], \tag{1.2}$$

where θ is the angle between the momentum of the positron and the spin of the muon and $\mathbb{P}_\mu$ is the polarization of the muon. Notice that integrating over $\cos\theta$, we arrive at $\Gamma(\mu^+ \to e^+\gamma) = (m_\mu^3/4\pi)(|A_L|^2 + |A_R|^2)$. Thus, by measuring the total decay rate to $e^+\gamma$, we can only measure $|A_L|^2 + |A_R|^2$. However, eq. (1.2) shows that by studying the angular distribution of the final particles with a moderate angular resolution, $|A_R|^2$ and $|A_L|^2$ can be separately derived. Information on $|A_L|^2/|A_R|^2$ can be translated into information on the sources of lepton flavor violation in the underlying theory. Studying the angular distribution can therefore be considered as a tool to discriminate between different scenarios beyond the SM [4, 3]. Moreover, in case of low statistics, studying the angular distribution can help us to veto the background [4]. Notice, however, that with this method only the absolute values of A_L and A_R can be derived and no information on the relative phase of A_L and A_R can be extracted. Whereas the relative phase of A_L and A_R carry valuable information on the sources of CP-violation in the underlying theory. In this paper, we show that if in addition to the angular distribution of the final particles in the LFV muon decay, we also measure their polarization, we will be able to extract the phase of $A_L^* A_R$. Remembering the fact that the state-of-the-art LHC experiment will most likely not be able to measure these phases and for measuring such phases a more advanced collider, ILC, is proposed [5], the possibility of measuring these phases by muon decay experiments seems more exciting. In section 2, we show that by studying the angular distribution of transversely polarized positrons and photons we can extract the relative phase of A_L and A_R. In section 3, we discuss the possibility of extracting the same information by studying the angular distribution of the final positrons produced in $\mu^+ \to e^+e^-e^+$. We then compare the two methods and discuss the advantages and disadvantages of each one. We summarize our results in section 4.

2. Lepton flavor violating rare decay $\mu \to e\gamma$

Consider an anti-muon at rest [i.e., $P_{\mu^+} = (m_\mu, 0, 0, 0)$] which decays into a positron and a photon with definite spins of $\vec{s}_e$ and $\vec{s}_\gamma$, respectively. Using the effective Lagrangian (1.1), we can calculate the $\mu \to e\gamma$ decay rate:

$$\begin{aligned} \frac{d\Gamma[\mu^+(P_{\mu^+}) \to e^+(P_{e^+}, \vec{s}_{e^+})\gamma(P_\gamma, \vec{s}_\gamma)]}{d\cos\theta} = \frac{m_\mu^3}{8\pi} \Big[& |\alpha_+|^2 |A_L|^2 (1 + \mathbb{P}_\mu \cos\theta) \sin^2\frac{\theta_s}{2} + \qquad (2.1) \\ & |\alpha_-|^2 |A_R|^2 (1 - \mathbb{P}_\mu \cos\theta) \cos^2\frac{\theta_s}{2} \\ & + \mathbb{P}_\mu \mathrm{Re}[\alpha_+ \alpha_-^* A_L^* A_R e^{i\phi_s}] \sin\theta \sin\theta_s \Big], \qquad (2.2) \end{aligned}$$

where $\mathbb{P}_\mu$ is the polarization of the anti-muon, θ is the angle between the directions of the spin of the anti-muon and the momentum of the positron, and θ_s is the angle between the spin of the positron and its momentum. In the above formula, ϕ_s is the azimuthal angle that the spin of the final positron makes with the plane of spin of the muon and the momentum of the positron (to be specific to measure ϕ_s, the coordinate system has been defined as follows: $\hat{z} = \vec{p_{e^+}}/|\vec{p_{e^+}}|$ and $\hat{y} = \vec{s}_{e^+} \times \hat{z}/|\vec{s}_{e^+} \times \hat{z}|$). Finally, α_+ and α_- give the polarization of the final photon: $\varepsilon^\mu = (0, \alpha_+ + \alpha_-, (\alpha_+ - \alpha_-)i, 0)/\sqrt{2}$ with $\sqrt{|\alpha_+|^2 + |\alpha_-|^2} = 1$.

Summing over the spins of the final particles, we arrive at the well-known formula shown in eq. (1.2) which does not contain any information on the relative phase of A_L and A_R. Moreover, from eq. (2.1) it is clear that in order to be sensitive to the phase of $A_L A_R^*$ the combination $\alpha_+ \alpha_-^* \sin\theta_s$ should be nonzero. Remember that $\alpha_- = 0$ and $\alpha_+ = 0$ respectively correspond to positive and negative helicities. On the other hand, $\sin\theta_s = 0$ corresponds to either a right-handed positron (for $\theta_s = 0$) or to a left-handed positron (for $\theta_s = \pi$). Thus, in order to extract the *relative phase of* A_L *and* A_R from $\mu^+ \to e^+\gamma$ we have to study the final positrons and photons whose spins are not parallel to their momenta.

Let us now consider the CP conjugate of the same process. It is straightforward to prove that the partial decay rate of the CP conjugate process, $d\bar{\Gamma}/d\cos\theta$, is given by (2.1) replacing $A_L \to A_L^*$ and $A_R \to A_R^*$. In other words, we obtain

$$\frac{d\Gamma}{d\cos\theta} - \frac{d\bar{\Gamma}}{d\cos\theta} = \frac{m_\mu^3}{4\pi}\mathbb{P}_\mu \mathrm{Im}[\alpha_+\alpha_-^* e^{i\phi_s}]\mathrm{Im}[A_L A_R^*]\sin\theta\sin\theta_s. \tag{2.3}$$

As expected the difference is given by the imaginary part of $A_L A_R^*$. Eq. (2.1) shows that if we can run the experiment both in the muon and anti-muon modes, we will be able to derive $\mathrm{Im}[A_L A_R^*]$ even without studying the angular distribution of the final lepton:

$$\int \frac{d\Gamma}{d\cos\theta} d\cos\theta - \int \frac{d\bar{\Gamma}}{d\cos\theta} d\cos\theta = \frac{m_\mu^3}{8}\mathbb{P}_\mu \mathrm{Im}[\alpha_+\alpha_-^* e^{i\phi_s}]\sin\theta_s \mathrm{Im}[A_L A_R^*].$$

At first sight, it may seem that the above relation is at odds with the generalized optical theorem [6] which states that the *total* decay rate of a particle and an anti-particle should be equal. Notice, however that we have not summed over the final spins so the integrals on the left-hand side do not give the *total* rate of $\mu \to e\gamma$. In fact, the above equation shows that summing over the spin of the photon and/or the positron the difference vanishes, as expected from the generalized optical theorem. The effect is maximal for linearly polarized photons (i.e., $\alpha_- = \pm\alpha_+ = 1/\sqrt{2}$) and for the final leptons polarized in the direction perpendicular to the direction of the spin of muon and the momentum of the final lepton (i.e., $\theta_s = \pi/2, \phi_s = \pi/2$).

If we have only the anti-muon mode available (or only the muon mode available), we can still extract $\mathrm{Im}[A_L A_R^*]$ by studying the angular distribution of the final leptons. Notice that

$$\int_{-1}^{-1/2} \frac{d\Gamma}{d\cos\theta} d\cos\theta - \int_{-1/2}^{1/2} \frac{d\Gamma}{d\cos\theta} d\cos\theta + \int_{1/2}^{1} \frac{d\Gamma}{d\cos\theta} d\cos\theta = \frac{m_\mu^3}{8\pi}\mathbb{P}_\mu \left(\frac{\pi}{6} - \frac{\sqrt{3}}{2}\right) \times \\ \times \mathrm{Re}[\alpha_+\alpha_-^* e^{i\phi_s} A_L^* A_R]\sin\theta_s, \tag{2.4}$$

which shows that sensitivity to $\mathrm{Im}[A_L^* A_R]$ is maximal again for linearly polarized photons (i.e., $\alpha_- = \pm\alpha_+ = 1/\sqrt{2}$) and leptons polarized in the direction perpendicular to the direction of the spin of muon and the momentum of the final lepton (i.e., $\theta_s = \pi/2, \phi_s = \pi/2$).

Measuring the transverse polarization of the final lepton is feasible. In fact, this technique has long been employed to measure the Michel parameters [7]. Measuring the linear polarization of photon at energies of $\sim 50\,\mathrm{MeV}$ also seems practical [8]. As recently shown in [9] equipping the experiments with photon polarimeters can have implications for studying the radiative muon decay, too.

The phases of the underlying theory that manifest themselves in the LFV effective Lagrangian (1.1) can also induce a contribution to the electric dipole moment of the electron, d_e. In the following, we estimate the effect on d_e and compare it with the present bound.[1] We can write the effective couplings A_R and A_L in terms of the parameters of the underlying theory as follows

$$A_L \sim \frac{\lambda^2}{16\pi^2}\frac{(m^2_{\mu e})_L}{(m^2_{\rm NEW})^2} m_\mu \qquad A_R \sim \frac{\lambda^2}{16\pi^2}\frac{(m^2_{\mu e})_R}{(m^2_{\rm NEW})^2} m_\mu \,, \tag{2.5}$$

where λ and $m_{\rm NEW}$ are respectively the coupling and the scale of the new physics. The factor $16\pi^2$ in the denominators are the loop factors which appear when we integrate out the heavy states at the one loop level. $(m^2_{\mu e})_L$ and $(m^2_{\mu e})_R$ are the sources of LFV at the left and right-handed sectors, respectively. The presence of m_μ reflects the chirality flipping nature of the corresponding effective operators. (We have assumed that the relation between chirality-flipping and fermion mass is maintained in the framework of the new physics.) Finally, the power of $m_{\rm NEW}$ in the denominator is fixed by dimensional analysis. At the one loop level, the phases of $(m^2_{\mu e})_L$ and $(m^2_{\mu e})_R$ can induce a contribution to d_e which can be estimated as

$$d_e \sim e\frac{\lambda^2}{16\pi^2}\frac{{\rm Im}[(m^2_{\mu e})^*_L (m^2_{\mu e})_R]}{(m^2_{\rm NEW})^3} m_\mu \,.$$

Inserting A_L and A_R of eq. (2.5) in the above equation and using $\Gamma(\mu\to e\gamma) = m_\mu^3(|A_R|^2 + |A_L|^2)/(4\pi)$, we find

$$d_e = (10^{-32}\ e\ {\rm cm})\left(\frac{0.5}{\lambda^2}\right)\frac{{\rm Im}[A_L A_R^*]}{|A_L|^2+|A_R|^2}\left(\frac{m_{\rm NEW}}{100\ {\rm GeV}}\right)^2\frac{{\rm Br}(\mu\to e\gamma)}{10^{-11}}\,.$$

Thus, for supersymmetric models with $\lambda = g$ and $m_{\rm NEW} <$ few TeV the effect of these phases will be at least three orders of magnitude below the present bound [1] and too small to be probed even by forthcoming experiments [10]; that is while the method proposed in this paper can help us to derive information on the relative phase of A_L and A_R.

3. Three body decay $\mu^+ \to e^+e^-e^+$

The effective Lagrangian in eq. (1.1) can give rise to $\mu^+ \to e^+e^-e^+$ through the penguin diagrams shown in figure 1. The penguin diagrams are not the only diagrams that contribute to the decay mode $\mu^+ \to e^+e^-e^+$ (see the appendix for more details). However, we

[1] I would like to thank the anonymous referee for pointing out the relevance of this discussion.

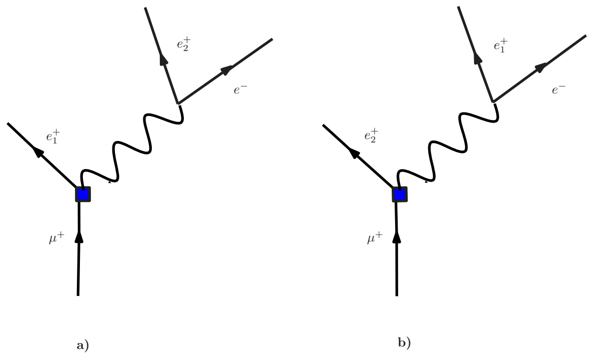

Figure 1: Penguin diagrams contributing to $\mu^+ \to e_1^+ e_2^+ e^-$. The vertices marked with boxes are the LFV vertices from interaction terms in eq. (1.1).

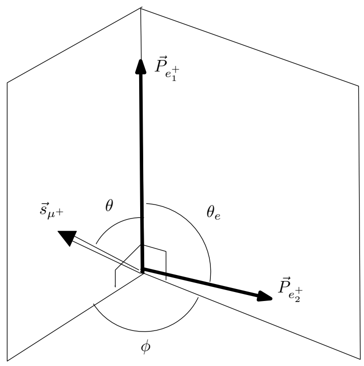

Figure 2: This figure schematically depicts the direction of the momenta of the final particles in the LFV decay $\mu^+ \to e_1^+ e^- e_2^+$ relative to the spin of the anti-muon in its rest frame.

generally expect the contribution from the penguin diagrams to be dominant because for the final lepton with energy close to $m_\mu/2$, as we will see momentarily, the photon in the penguin diagram can go almost on shell, resulting in an enhancement by $\ln(m_\mu/m_e)$. In this section, we will assume that the penguin diagrams shown in figure 1 are the only ones contributing to $\mu^+ \to e^+e^-e^+$. This is a reasonable approximation for the MSSM as well as a class of models which we describe in the appendix.

In fact, we expect about 90 % of the three-body $\mu^+ \to e^+e^-e^+$ decays to result in a lepton with energy $\simeq m_\mu/2$. In this section, we show that if we measure the spin of the final lepton with energy $m_\mu/2$ as well as the angular distributions of the final particles, we can extract information on the relative phase of A_L and A_R.

Consider an anti-muon at rest with a spin at the $(\hat{x}, \hat{z})$ plane which makes an angle of θ with the z-axis:

$$P_{\mu^+} = (m_\mu, 0, 0, 0) \qquad \bar{v}_{\mu^+} = \sqrt{m_\mu}(-\sin\frac{\theta}{2}, \cos\frac{\theta}{2}, \sin\frac{\theta}{2}, -\cos\frac{\theta}{2}), \tag{3.1}$$

where P_{μ^+} and v_{μ^+} are respectively the four-momentum and the Dirac spinor of the anti-muon. Suppose the anti-muon decays into an electron and two positrons with the following momenta:

$$P_{e_1^+} = (E_1, 0, 0, \sqrt{E_1^2 - m_e^2}) \quad P_{e_2^+} = (E_2, |\vec{P}_{e_2^+}| \sin\theta_e \cos\phi, |\vec{P}_{e_2^+}| \sin\theta_e \sin\phi, |\vec{P}_{e_2^+}| \cos\theta_e), \tag{3.2}$$

where $E_2^2 = |\vec{P}_{e_2^+}|^2 + m_e^2$. The above angles are illustrated in figure 2. The four-momentum of the final electron is determined by the energy-momentum conservation. The diagrams contributing to $\mu^+ \to e_1^+ e^- e_2^+$ are shown in figure 1. The amplitude corresponding to diagram 1-a can be written as

$$e\bar{u}_e \gamma^\nu v_{e_2^+} \frac{ig_{\nu\mu}}{q^2} \bar{v}_{\mu^+} \sigma^{\mu\alpha}(A_L P_R + A_R P_L) v_{e_1^+} q_\alpha, \tag{3.3}$$

where q_α is the four-momentum of the virtual photon in the penguin diagram: $q \equiv P_{\mu^+} - P_{e_1^+}$. Combining eqs. (3.1) and (3.2), we find $q^2 = m_\mu^2 + m_e^2 - 2E_1 m_\mu$. As a result, in the limit $E_1 \to m_\mu/2$, $q^2 \ll m_\mu^2$ and the amplitude in eq. (3.3) is considerably enhanced. That is while the propagator of the virtual photon appearing in figure 1-b is given by $1/(P_{\mu^+} - P_{e_2^+})^2 \sim 1/m_\mu^2$ so, in the limit $E_1 \to m_\mu/2$, the effect of diagram 1-b can be neglected in comparison to that of diagram 1-a. Moreover, in this limit, the effects of the LFV terms other than the term in eq. (1.1) are lower at least by a factor of m_e^2/m_μ^2 and can be also neglected (see the appendix for more details). Let us define $d\Gamma^{\rm Max}/d\cos\theta d\phi$ as partial decay rate of μ^+ into a positron with energy close to $m_\mu/2$ and spinor $v_{e_1^+} = \sqrt{2E_1}(0, d_e, c_e, 0)^T$:

$$\frac{d\Gamma^{\rm Max}}{d\cos\theta d\phi} = \sum_{\rm spins} \int_{m_\mu/2-\Delta E}^{E_{\rm max}} \int_{m_\mu/2-E_1}^{m_\mu/2} \frac{d\Gamma}{dE_2 dE_1 d\cos\theta d\phi} dE_2 dE_1, \tag{3.4}$$

where $\Delta E \ll m_\mu$ and $E_{\rm max} \simeq m_\mu/2 - 4m_e^2/m_\mu$. Notice that we have integrated and summed over the energies and spins of the pair of e_2^+ and e^- but not over those of e_1^+. It is straightforward to show that

$$\frac{d\Gamma^{\rm Max}}{d\cos\theta d\phi} = \frac{\alpha m_\mu^3}{192\pi^3} \Big[|A_L|^2 |c_e|^2 (1 + \mathbb{P}_\mu \cos\theta) + |A_R|^2 |d_e|^2 (1 - \mathbb{P}_\mu \cos\theta) \tag{3.5}$$
$$+ \mathbb{P}_\mu \sin\theta \left(\cos(2\phi) {\rm Re}[A_R A_L^* d_e c_e^*] + \sin(2\phi) {\rm Im}[A_R A_L^* d_e c_e^*]\right)\Big] \log \frac{m_\mu \Delta E}{4m_e^2},$$

where, as shown in figure 2, θ is the angle between the spin of the anti-muon and the momentum of e_1^+ and ϕ is the azimuthal angle of the momentum of e_2^+ measured from the plane made by the spin of μ^+ and the momentum of e_1^+ [see eq. (3.2)].

After integrating over ϕ and $\cos\theta$ and summing over the spin of e_1^+ (i.e., summing over states $c_e = 1,\ d_e = 0$ and $d_e = 1,\ c_e = 0$), we will arrive at the familiar formula in the literature (e.g., see [11]). However, in this case the information on the phase of $A_R A_L^*$ will be lost. In order to extract this phase, we have to be able to measure the spin of e_1^+ as well as the direction of the momenta of the final states relative to the spin of the anti-muon. Let us now define the following ratio

$$\mathcal{R} = \frac{\int_{-1}^{+1} d\cos\theta [\int_0^{2\pi} \frac{d\Gamma^{\rm Max}}{d\cos\theta d\phi} {\rm sgn}(\tan\phi) d\phi]}{\int_{-1}^{+1} d\cos\theta [\int_0^{2\pi} \frac{d\Gamma^{\rm Max}}{d\cos\theta d\phi} {\rm sgn}(\tan(\phi + \pi/4)) d\phi]}. \tag{3.6}$$

Notice that sgn($\tan\phi$) in the integral is equal to ± 1 depending on the quadrant that ϕ belongs to. In principle, if the polarization of the anti-muon is large (*i.e.,* $\mathbb{P}_\mu$ is about 100%), this ratio can be measured in the lab. Using eq. (3.5), we can show that

$$\mathcal{R} = \frac{\mathrm{Im}[A_R A_L^* d_e c_e^*]}{\mathrm{Re}[A_R A_L^* d_e c_e^*]},$$

which directly gives the phase of $A_R A_L^*$ for a transversely polarized positron, $d_e = c_e = 1/\sqrt{2}$.

Now let us compare the advantages and disadvantages of each decay mode. In general, we expect

$$\frac{\mathrm{Br}(\mu^+ \to e^+ e^+ e^-)}{\mathrm{Br}(\mu^+ \to e^+ \gamma)} \simeq \frac{\alpha}{3\pi}\left[\log\left(\frac{m_\mu^2}{m_e^2}\right) - \frac{11}{4}\right] \simeq 0.0061.$$

Thus, measurement of $\mu^+ \to e^+ e^- e^+$ will suffer from a higher statistical uncertainty. On the other hand, to extract the relative phase of A_L and A_R by studying $\mu^+ \to e^+\gamma$ in addition to measuring the spin of e^+, it is necessary to measure the spin of γ, too. Whereas in the case of $\mu^+ \to e^+ e^- e^+$, one has to measure only the spin of the final positron with energy close to $m_\mu/2$.

As is well-known in the case of a three-body decay mode, we can have CP- and T-odd observable quantities, even if the spins of the final particles are averaged over. However, the above discussion shows that *if* the effective Lagrangian (1.1) is the *only* source of LFV, once we average over the final spins, the CP- and T-odd effects will disappear. In fact, as shown in [12, 13], if the four-fermion LFV terms listed in the appendix are also present, the CP- and T-odd effects will persist even after averaging over the final spins. However, we generally expect these effects to be suppressed roughly by a factor of $C_i m_\mu/[A_{L,R}\log(4m_e^2/m_\mu \Delta E)]$ compared to the effect we have discussed in the present paper. Notice that the two effects are sensitive to different combinations of the CP-violating phases and can be thus considered as complementary.

4. Concluding remarks

In this paper, we have suggested a new method to derive information on the sources of CP-violation beyond those in the SM. The method is based on studying the rare LFV decay of polarized muons. We have performed our analysis within a general effective LFV Lagrangian so our results apply to any beyond SM scenario that violates the lepton flavor by adding new particles at energies higher than the electroweak symmetry breaking scale (*e.g.,* supersymmetry, large extra dimensions).

We have first studied the LFV rare decay $\mu \to e\gamma$ and shown that provided that the polarization of the final particles are not parallel to their momentum (e.g., if e and γ are respectively transversely and linearly polarized), by studying the angular distribution of e and the photon relative to the polarization of μ, we can extract information on the CP-violating phases. We have also shown that if both muon and anti-muon modes are available, the same information can be derived by comparing $\Gamma(\mu^+ \to e^+_{\nparallel}\gamma_{\nparallel})$ and $\Gamma(\mu^- \to e^-_{\nparallel}\gamma_{\nparallel})$ where the subscript $\nparallel$ indicates that the spin of the particle is not parallel to its momentum.

We have estimated d_e induced by the CP-violating phases that the present method aims to measure and found that the effect is at least three orders of magnitude below the present bound. In other words, the phases of order of $\pi/2$ associated with the μe mixing are not ruled out by the present bound on d_e.

We have also studied the $\mu^+ \to e_1^+ e^- e_2^+$ decay where e_1^+ is defined to be the more energetic positron. The amplitude of $\mu^+ \to e_1^+ e^- e_2^+$ is severely enhanced if the energy of e_1^+ is close to $m_\mu/2$. Thus, we expect the majority of e_1^+ to have energies close to $m_\mu/2$. We have focused on decays with such kinematics and proposed a new method for extracting information on the CP-violating phases which is based on studying the angular distribution of the final particles. We have shown that with transversely polarized e_1^+ one can extract information on a combination of the CP-violating phases that is impossible to achieve if e_1^+ with helicity ± 1 is employed or if the final spins are averaged over. Notice that in this method measuring the spin of only one of the final particles (i.e., e_1^+) will be enough. We have discussed the differences and synergies between this method and the one discussed in [12, 13].

A. LFV effective Lagrangian

In the appendix, we discuss possible LFV operators that appear by integrating out the heavy states within theories such as the MSSM and show that the effect of eq. (1.1) on the rare LFV muon decays is dominant.

In the literature (see e.g., [11]), it has been shown that in the context of the MSSM, integrating out the heavy supersymmetric states the effective LFV Lagrangian of the electron-muon system will, in addition to eq. (1.1), contain

$$\varepsilon^\alpha \bar{\mu} q^2 \gamma_\alpha (B_L P_L + B_R P_R) e + \text{H.c.}, \tag{A.1}$$

where ε^α is the photon field, q is the momentum of the photon, P_L (P_R) is left (right) projection matrix and B_L and B_R are couplings with dimension of $[\text{mass}]^{-2}$. This effective term will have no impact on $\mu \to e\gamma$ simply because for on-shell photon ($q^2 = 0$), this term vanishes. However, in general it can contribute to $\mu^+ \to e^+ e^- e^+$ through a penguin diagram. Notice that unlike the case of eq. (3.3), in this case the penguin diagram does not diverge as the photon propagator goes on-shell. Moreover, for most of the parameter space of the MSSM $B_{L,R} m_\mu \ll A_{L,R}$ so the effect of eq. (A.1) is further suppressed. As a result, for calculating Γ^{Max} [defined in eq. (3.4)] we can neglect the effect of eq. (A.1).

The effective LFV effective Lagrangian will also contain the following four-fermion terms that can in principle contribute to $\mu^+ \to e^+ e^- e^+$:

$$\begin{aligned}\mathcal{L} = \; & C_1(\bar{\mu}_R e_L)(\bar{e}_R e_L) + C_2(\bar{\mu}_L e_R)(\bar{e}_L e_R) \\ & + C_3(\bar{\mu}_R \gamma^\mu e_R)(\bar{e}_R \gamma_\mu e_R) + C_4(\bar{\mu}_L \gamma^\mu e_L)(\bar{e}_L \gamma_\mu e_L) \\ & + C_5(\bar{\mu}_R \gamma^\mu e_R)(\bar{e}_L \gamma_\mu e_L) + C_6(\bar{\mu}_L \gamma^\mu e_L)(\bar{e}_R \gamma_\mu e_R) + \text{H.c.}\end{aligned} \tag{A.2}$$

Again, we expect that the effect of the above four-fermion terms on $\Gamma^{\max}$ [see eq. (3.4)] to be negligible compared to the terms in eq. (1.1). That is because, unlike the penguin diagrams in figure 1, the diagrams corresponding to the above four-fermion interaction terms do not diverge for $E_1 \to m_\mu/2$. Moreover, for the major part of the parameter space of the MSSM (with $\tan\beta \sim 10$) the four-fermion effective couplings are smaller than the dipole couplings; i.e., $C_i m_\mu/A_{L,R} \sim 1/\tan\beta$.

Consider a general scenario in which new particles with mass $m_{\rm NEW} \gg m_\mu$ are added to the SM. Suppose that all LFV as well as LF conserving couplings of the new particle with the SM particles are of order of λ. Within such a scenario we can write

$$A_L, A_R \sim \frac{\lambda^2}{16\pi^2}\frac{m_\mu}{m_{\rm NEW}^2} \quad \text{and} \quad C_i \sim \frac{\lambda^4}{16\pi^2}\frac{1}{m_{\rm NEW}^2} \; .$$

Notice that here, unlike the case of (2.5), we have assumed that the suppression of $\mathrm{Br}(\mu \to e\gamma)$ is due to the smallness of the couplings (i.e., $\lambda^2/(4\pi) \ll \alpha$) rather than small mixing [i.e., $(m_{e\mu}^2)_{L,R}/m_{\rm NEW}^2 \ll 1$]. The bound on $\mathrm{Br}(\mu \to e\gamma)$ can be translated into a bound on

$$\lambda \sim 10^{-3}\left(\frac{m_{\rm NEW}}{100\,GeV}\right)\sqrt[4]{\mathrm{Br}(\mu \to e\gamma)/10^{-11}}.$$

Thus, the contribution of the four-fermion terms to the amplitude of $\mu^+ \to e^+e^-e^+$ in comparison to that of the couplings in (1.1) is further suppressed by a factor of $m_\mu C_i/[(4\pi\alpha)A_{L,R}] \sim \lambda^2/(4\pi\alpha) \lesssim 10^{-4}$.

One should notice that the most general effective LFV Lagrangian, in addition to the terms discussed above, contains extra terms. For example, it is possible to have terms such as

$$\epsilon^{\alpha\beta\mu\nu}\bar{\mu}p_\alpha\gamma_\beta(D_L P_L + D_R P_R)eF_{\mu\nu} + \text{H.c.},$$

where p_α is the four-momentum of the electron. However, studying the effective LFV Lagrangian in its most general form is beyond the scope of this paper.

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